ii) Viscous forces which in turn depend upon the axial velocity gradient.

The first factor should increase the pressure drop with increases in number of bends while the second factor tends to reduce it owing to the weaker velocity gradients caused by interchange of velocities at the bends. For fewer bends ($n \leqslant 2$), the first factor is less effective but the second one shows its substantial effect (reflected by the shifting of θ_{min} from 0.613 to 0.68 even for a single bend, Figure 2) causing a reduction in pressure drop. As the number of bends is increased ($n > 2$), the first factor dominates which enhances pressure drop. The maximum enhancement in friction factor due to bending the coils (with 57 bends) was found to be about 1.7 fold at a Dean number of about 35.

The ease of fabrication, compactness and narrower RTD found with coiled flow inverters establish their superiority over other mechanical devices, known in literature, for inducing mixing in a cross-sectional plane and making flow closer to plug flow.

NOTATION

D = effective diffusion coefficient, cm^2/s
d_c = diameter of the helical coil, cm
d_{cc} = diameter of the coiled coil, cm
d_t = tube diameter, cm
F = residence time distribution function, dimensionless
l = length of the tube, cm
n = number of bends
N_{De} = Dean number ($= N_{Re}/\sqrt{\lambda}$), dimensions
N_{Re} = Reynolds number ($= d_t\bar{u}\rho/\mu$), dimensionless
R_A = parameter defined by Eq. 1
$\bar{t}$ = mean holding time, s
$\bar{u}$ = overall average axial velocity, cm/s
θ = dimensionless residence time
θ_{min} = residence time for the fastest moving fluid element, dimensionless
λ_c = coil to tube diameter ratio ($= d_c/d_t$), dimensionless
λ_{cc} = coiled coil to coil diameter ratio, dimensionless
ρ = fluid density, g/cc
μ = fluid viscosity, cp

LITERATURE CITED

Danckwerts, P. V., "Distribution of Residence Time," *Chem. Eng. Sci.*, **2**, p. 1 (1953).
Dean, W. R., "Note on the Motion of Fluid in Curved Pipes," *Phil. Mag.*, **4**, p. 207 (1927).
Erdogan, M. E., and P. C. Chatwin, "The Effect of Curvature and Buoyancy on the Laminar Dispersion of Solute in a Horizontal Tube," *J. Fluid Mech.*, **29**, p. 465 (1967).
McConalogue, D. J., "The Effect of Secondary Flow on the Laminar Dispersion of an Injected Substance in a Curved Tube," *Proc. Roy Soc.*, **A315**, p. 99 (1970).
McConalogue, D. J., and R. S. Srivastava, "Motion of a Fluid in a Curved Tube," *Proc. Roy. Soc.*, **A307**, p. 37 (1968).
Mishra, P., and S. N. Gupta, "Momentum Transfer in Curved Pipes 1. Newtonian Fluids," *Ind. Eng. Chem. Proc. Des. Dev.*, **18**, p. 130 (1979).
Mujavar, B. A., and M. Raja Rao, "Flow of Non-Newtonian Fluids Through Helical Coils," *Ind. Eng. Chem. Proc. Des. Dev.*, **17**, p. 22 (1978).
Nauman, E. B., "The Residence Time Distribution for Laminar Flow in Helically Coiled Tubes," *Chem. Eng. Sci.*, **32**, p. 287 (1977).
Nauman, E. B., "Enhancement of Heat Transfer and Thermal Homogeneity With Motionless Mixers," *AIChE J.*, **25**, p. 246 (1979).
Nigam, K. D. P., and K. Vasudeva, "Studies on Tubular Flow Reactor With Motionless Mixing Elements," *Ind. Eng. Chem. Proc. Des. Dev.*, **15**, p. 473 (1976).
Nigam, K. D. P., and K. Vasudeva, "Residence Time Distribution in Static Mixers," *Cand. J. Chem. Eng.*, **58**, p. 543 (1980).
Nunge, R. J., T. S. Lin, and W. N. Gill, "Laminar Dispersion in Curved Tubes and Channel," *J. Fluid Mech.*, **51**, p. 363 (1972).
Ruthven, D. M., "The Residence Time Distribution for Ideal Laminar Flow in a Helical Coil," *Chem. Eng. Sci.*, **26**, p. 1113 (1971).
Saxena, A. K., "Laminar Dispersion in Helically Coiled Tubes," Ph.D. Thesis, Indian Institute of Technology, Delhi, India (1982).
Saxena, A. K., and K. D. P. Nigam, "On RTD in Coiled Tubes," *Chem. Eng. Sci.*, **34**, p. 425 (1979).
Taylor, G. I., "Dispersion of Soluble Matter in Solvent Flowing Slowly Through a Tube," *Proc. Royal Soc.*, **219 A**, p. 186 (1953).
Trivedi, R. N., and K. Vasudeva, "RTD for Diffusion Free Laminar Flow in Helical Coils," *Chem. Eng. Sci.*, **29**, p. 2291 (1974).
Trivedi, R. N., and K. Vasudeva, "Axial Dispersion in Laminar Flow in Helical Coils," *Chem. Eng. Sci.*, **30**, p. 317 (1975).

Manuscript received November 18, 1982; revision received March 31, and accepted April 5, 1983.

Dynamics and Control of an Activated Sludge Process As a Mixed Culture System

TAKEICHIRO TAKAMATSU, SUTEAKI SHIOYA, and HIROKAZU KUROME

Department of Chemical Engineering
Kyoto University
Kyoto, Japan

One of the difficulties encountered in the operation of an activated sludge process is the phenomenon of bulking sludge.

In an activated sludge process which is composed of a completely mixed aeration tank and a sedimentation vessel, the dynamic behavior of the system can be analyzed using a mathematical model. The model developed here is based on the kinetics and settleability of the combination of floc-forming sludge and bulking sludge. The operating conditions that cause the bulking phenomenon are clarified on the phase plane. It is also shown that a type of nonlinear state feedback regulator makes the system stable.

SCOPE

An activated sludge process is used for the biological oxidation of sewage. In its conventional form it consists of an aerated reactor and a sludge sedimentation tank which provides recycled sludge. One of the difficulties encountered in the operation of the activated sludge process is connected with the phenomenon of bulking sludge. Bulking sludge settles poorly

Correspondence concerning this paper should be addressed to S. Shioya.

and does not compact well. Consequently, activated sludge is lost over the effluent weir of the sedimentation tank, the concentration of the return sludge is reduced, and the process finally breaks down. The prevention and control of the bulking phenomenon are therefore of considerable importance. Most frequently, bulking sludge shows the presence of an excess number of filamentous organisms (Rensink, 1974) which are for the most part completely different from a normal sludge in which floc-forming microorganisms are dominant.

The tank's mode of operation in terms of organic loarding, dissolved oxygen(DO) levels, PH and temperature influence the physical properties of the activated sludge. In this paper, the main concern is the effect of the organic loading on the bulking phenomenon of the activated sludge. The objectives of this work are to develop a simple mathematical model describing the bulking phenomenon caused by high sludge-loading and to develop some means of preventing and controlling bulking sludge. The model system selected for examination was a mixed culture of two groups of microorganisms, one having the property of floc-forming called "floc-forming sludge" and the other being "bulking sludge."

The growth kinetics in the aeration tank and the settleability of the activated sludge in the sedimentation vessel were studied experimentally and modeled. In an activated sludge process composed of a completely mixed aeration tank and a sedimentation tank, the dynamic behavior of the system was analyzed using a mathematical model. This model simulates both the kinetics and settleability of the two populations of organisms, namely the floc-forming and bulking sludge populations.

Stephanopoulos (1980) presented a theoretical study of some topological aspect of the dynamics of mixed cultures of microorganisms and presented general results in some typical mixed-culture systems with no recycled cells. His study utilized the degree theory and Hopf's index theorem. In this paper, the interaction of two populations which compete for a single rate-limiting substrate with a cell recycle stream is analyzed using phase plane analysis. The above technique of using computer simulations on the phase plane was used to understand two aspects of the system: 1) the way in which the operating and input conditions, including flow rate of recycled sludge and the flow rate and COD concentration of the influent, affect the stability and dynamic behavior of the system; and 2) the conditions that cause and prevent the bulking phenomenon.

CONCLUSIONS AND SIGNIFICANCE

The growth kinetics of two groups of microorganisms in two types of synthetic media, one in which the carbon source are glucose and the other in which the carbon sources are glucose and polypeptone, were studied experimentally using batch cultures, where the effect of the organic loading rate on the bulking phenomenon was main subject. The other factors including DO levels were kept high enough so that they do not become a limiting factor in the bulking phenomenon. In this system, the COD (Chemical Oxygen Demand) concentration S, is considered to be that of the limiting substrate. The significant results for the obtained growth kinetics are: 1) there is no direct interaction between the two groups of microorganisms, but there is competition between the two groups which grow utilizing a common substrate S; 2) the specific growth rates of floc-forming sludge and bulking sludge are both described by the increasing linear relationships with the COD concentration in the region of COD which is low (less than 200 $g \cdot m^{-3}$); and 3) for one value of the COD concentration, specific growth rate lines intersect.

From the study of the settleability of the activated sludge, the concentration of the recycled sludge can be related to the ratio of the amount of bulking sludge against floc-forming sludge utilizing experimental data.

Dynamic behavior of an activated sludge process, composed of a completely mixed aeration tank, and a sedimentation vessel, due to changing the sludge loading with the cell recycle stream was analyzed utilizing phase plane analysis. It was shown that there are, in general, four equilibrium points; normal state $E-1(X_F \neq 0, X_B = 0)$, bulking state $E-2(X_F = 0, X_B \neq 0)$, coexistence state $E-3(X_F \neq 0, X_B \neq 0)$ and washout state $E-4(X_F = 0, X_B = 0)$ where X_F is the concentration of the floc-forming sludge, and X_B is the concentration of the bulking sludge. It was verified that the coexistent state $E-3$ is unstable under any operating conditions. The domain of deviation that will make the normal state $E-1$ unstable in the spaces of the operation of sludge loading and in the initial conditions was clarified. This was also verified experimentally.

The system was transferred to the bulking state $E-2$ with no-control; nevertheless, it was shown that a type of nonlinear state feedback regulator made the system stable, that is, maintained the system at the nominal state $E-1$. The nonlinear feedback regulator was constructed by the nonlinear transformation of a linear regulator. The result shows that the nonlinear regulator works very well and that the normal state where floc-forming sludge is dominant can be maintained well.

INTRODUCTION

The overgrowth of activated sludge flocs by filamentous microorganisms results in a phenomenon known as "bulking." It is a major problem in the operation of activated sludge plants. Different opinions exist concerning the cause of bulking (Verachtert et al., 1980); Waste water composition, sludge loading, dissolved oxygen concentration, the feeding pattern of the aerator and PH are some of the factors reported (Chudova et al., 1973a, 1974). It was also reported that a low level of DO in the aeration tank was one of major factors of bulking (Palm et al., 1980).

With respect to sludge loading, many authors mentioned that for any waste a certain range of loading exists to obtain good settling sludges. An increase in the loading might then result in an increase of the sludge volume index (Ford and Eckenfelder, 1976). Chudova et al. (1973b) assumed that filamentous bacteria had a lower K_S (substrate affinity constant) than floc-forming bacteria which would favor their growth in completely mixed systems. However, they did not prove this theory. Moreover, the theory did not explain the growth of filamentous organisms in plug flow systems at high sludge loading and lower sludge ages (Chudova et al., 1974).

Settleability models for activated sludge in the sedimentation vessel related to the ratio of bulking sludge have seldom been reported (Dick, 1970; Attir and Denn, 1978). However, a mathematical model of the settleability of activated sludge is required in order to analyze an activated sludge process composed of an aeration tank and sedimentation vessel using the stability theory and phase plane analysis.

The objective of this work was to develop a simple mathematical model describing the bulking phenomenon caused by high sludge loading, including the growth kinetics of the aeration tank and the settleability of the sludge and to obtain some means of preventing and controlling bulking sludge. In our experiment, only the sludge loading rate was changed, and the other operating conditions, e.g., PH and temperature were kept constant. The DO concentration in the aeration tank was maintained at a high level so as not to cause bulking phenomenon. The model system selected for examination is a mixed culture of two groups of microorganisms, one has the property of floc-forming, called "floc-forming sludge," and the other is "bulking sludge."

A continuous flow biochemical reactor, in which a mixed culture of microorganisms is grown, very often exhibits a multiple of steady states. The analysis of the stability of these steady states has been the subject of research. Diviasio et al. (1978, 1981) reported the existence of multiple stable and unstable steady states in a CSTBR from theoretical and experimental studies. Several researchers (Aris and Humphrey, 1977; Yoon and Blanch, 1977; Yoshida et al., 1979; Stephanopoulos et al., 1979; Stephanopoulos, 1980) reported on the dynamic behavior and stability of two or more microorganism-mixed culture systems. The techniques most frequently employed in determining the stability characteristics of a steady state are based on Liapunov's first method (Athans and Falb, 1966). It consists mainly of a linearization of the differential equations around the steady state and an examination of the eigenvalue of the Jacobian matrix.

Stephanopoulos (1980) reported a theoretical study of some topological aspects of the dynamics of mixed cultures of microorganisms. Here, the dynamic behavior of the activated sludge process is analyzed based on a mathematical model utilizing phase plane analysis and computer simulation. Stephanopoulos (1980) analyzed a special case of the general type of mixed cultures of microorganisms. The results including the condition that leads the system to the bulking steady state will be very informative and useful for the practical operation of the activated sludge process.

EXPERIMENTAL METHODS AND RESULTS

It is assumed that activated sludge processes are mixed culture systems of two groups of microorganisms, one having the property of floc-forming called "floc-forming sludge," and the other being "bulking sludge." The kinetics of these two groups of microorganisms and the settleability of these sludges were studied experimentally and were modeled.

The kinetics in the two types of synthetic media, one in which the carbon source is glucose (medium-B) and the other in which the carbon source is glucose and polypeptone (medium-A), were studied experimentally using batch cultures. In this system, the COD (Chemical Oxygen Demand) concentration S, is considered to be that of the limiting substrate.

Culture Media and Method

Both populations, that is, the floc-forming and the bulking sludges, were cultured by feeding different types of synthetic wastewater on a fill-and-draw basis. Two kinds of synthetic media were used to purify and maintain each group of microorganisms. The composition of the standard medium-A was: 0.2 $kg{\cdot}m^{-3}$, polypeptone; 0.2 $kg{\cdot}m^{-3}$, glucose; 0.02 $kg{\cdot}m^{-3}$, KH_2PO_4. The solution had a COD concentration of about 220 $g{\cdot}m^{-3}$. In every case, the polypeptone:glucose:KH_2PO_4 ratio was kept at 1:1:0.1 in medium-A. The composition of the standard medium-B was: 0.6 $kg{\cdot}m^{-3}$, glucose; 0.15 $kg{\cdot}m^{-3}$, $(NH_4)_2SO_4$; 0.03 $kg{\cdot}m^{-3}$, KH_2PO_4. The solution had a COD concentration of about 900 $g{\cdot}m^{-3}$. The glucose:$(NH_4)_2SO_4$:KH_2PO_4 ratio was always kept at 20:5:1 in medium-B.

The initial sludge used consisted of domestic sludge withdrawn from the aeration basin of the Toba sewage treatment plant. The sludge was fed for a long period of time, more than one month, with synthetic medium-A or -B on a fill-and-draw basis. The temperature was kept around 22°C by a thermoregulator. Tanks 0.05 m^3 in volume were used, and 0.025 m^3 of mixed liquor was replaced with a fresh medium every day. The oxygen supply was sufficient enough for the microorganisms. Using medium-A in which the initial COD concentration was about 110 $g{\cdot}m^{-3}$ and the organic loading was from 0.05 to 0.1 kg-COD/kg-MLSS/d, the sludge became well-settling after less than 10 days from the start. This sludge is called "floc-forming sludge."

On the other hand, using medium-B in which the initial COD concentration was about 450 $g{\cdot}m^{-3}$ and organic loading was from 0.5 to 1.0 kg-COD/kg-MLSS/d, the sludge became an ill-settling sludge, that is, with a high sludge volume index (SVI) less than 4 days after the start. The sludge could be kept stable for more than one month by feeding medium-B. This sludge was called "bulking sludge."

Using the microscope it is observed that "bulking sludge" was composed

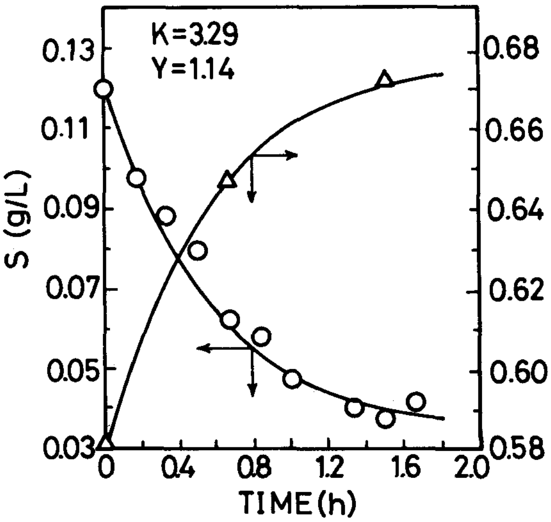

Figure 1. The batch culture of floc-forming sludge using medium-A; ○; COD concentration [$kg{\cdot}m^{-3}$], △; cell concentration [$kg{\cdot}m^{-3}$].

of filamentous microorganisms (e.g., *Sphaphaerotilus sp.*), while floc-forming sludge did not contain such microorganisms. Sometimes, *Volticella sp.* is observed in the floc-forming sludge.

Kinetics of COD Uptake and Cell-Growth

The fermenter used was a normal 0.001 m^3, equipped with automatic temperature control, an air flow rate indicator, DO sensors, and a magnetic agitator. The COD concentration and dry cell weight were measured during the experiments. The kinetics of floc-forming and bulking sludges were analyzed from the batch experiments using medium-A or -B. The initial COD concentration was changed depending upon the sludge concentration, and floc-forming and bulking sludges cultured by the method stated above were used for the experiments. Data for the initial 30 minutes were omitted from the analysis of the kinetics because initial bioabsorption occurred during this period.

One of the results obtained is shown in Figure 1 which shows the time courses of concentration of floc-forming sludge X_F and the COD concentration S with medium-A. From this figure, it was shown that the specific growth rate μ_F and the COD uptake rate ν_F could be represented by the linear relationship with respect to COD concentration S and that the yield factor was constant. Similar results were obtained in both types of sludge and in both types of media. Their mathematical forms are given as:

$$\begin{aligned} \mu_j &= K_j(S - S_j) \\ \nu_j &= \frac{-1}{Y_j}\mu_j \end{aligned} \qquad j = F, B \qquad (1)$$

where suffixes F and B mean floc-forming sludge and bulking sludge, respectively. $S_j(j = F, B)$ is a constant parameter.

The parameters were estimated, Table 1, by the least-square method based on the experimental data. The solid lines in Figure 1 show the calculated values using these parameters. The kinetic model described by Eq. 1 will be accepted in general in the range of low COD concentration.

Settleability

Settleability of the activated sludge in the sedimentation vessel was also studied experimentally. Unfortunately, a mathematical model describing the settleability of the sludge related to bulking sludge could not be found in the literature. Only some data were reported (Unno et al., 1981). The

TABLE 1. PARAMETERS ESTIMATED USING EXPERIMENTAL DATA

	Medium	$K_j(j = F, B)$	$Y_j(j = F, B)$	$S_j(j = F, B)$
Floc-Forming Sludge $j = F$	A	3.29	1.14	0.035
	B	3.38	1.29	0.005
Bulking Sludge $j = B$	A	4.15	1.29	0.045
	B	6.38	1.30	0.035

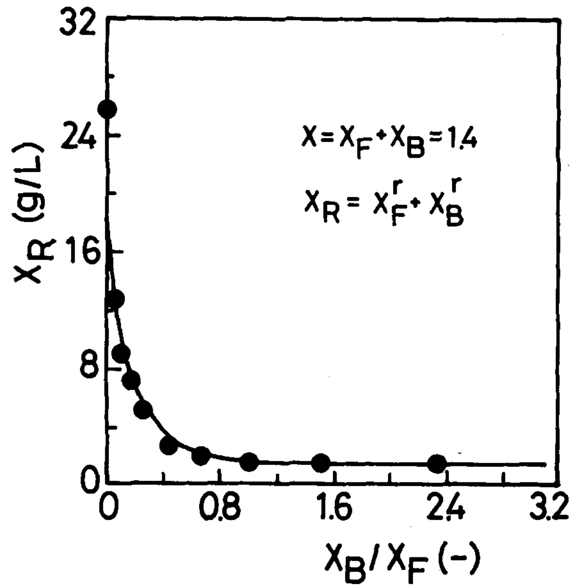

Figure 2. The relationship between the concentration of settled sludge and the ratio of bulking sludge to floc-forming sludge X_B/X_F.

empirical description of the concentration of the sedimenting sludge related to the ratio of X_F/X_B was obtained by the following experiments. First, both sampling liquids which contain floc-forming or bulking sludge cultures, as stated above, were adjusted so as to be of the same concentration. Then, a sampling liquid containing floc-forming sludge and one containing bulking sludge were mixed at a certain ratio and settled in a 0.001 m^3 cylinder. After 30 minutes, the sludge volume (SV30) was measured. From SV30 and the adjusted concentration of the sludge, the concentration of the settled sludge was easily calculated because the solid content in the supernatant liquor was certified to be almost zero.

Figure 2 shows the experimental result of the relationship between the concentration of the settled sludge, X_R, and the ratio of the bulking sludge to the floc-forming sludge, X_B/X_F. The relationship can be described by the power series of (X_B/X_F). From the literature (Unno et al., 1980), the same relationship is verified. The ratio of the concentration of the settled sludge, X_R, against the concentration of the sludge to be settled, X which is denoted by ξ can be represented by,

$$\xi \triangleq \frac{X_R}{X} = \alpha\left(\frac{X_B}{X_F} + \beta\right)^{-\gamma} + 1 \qquad (2)$$

The solid line is the calculated value using the estimated parameter: $\alpha = 71.4$, $\beta = 1.3$, $\gamma = 7.0$.

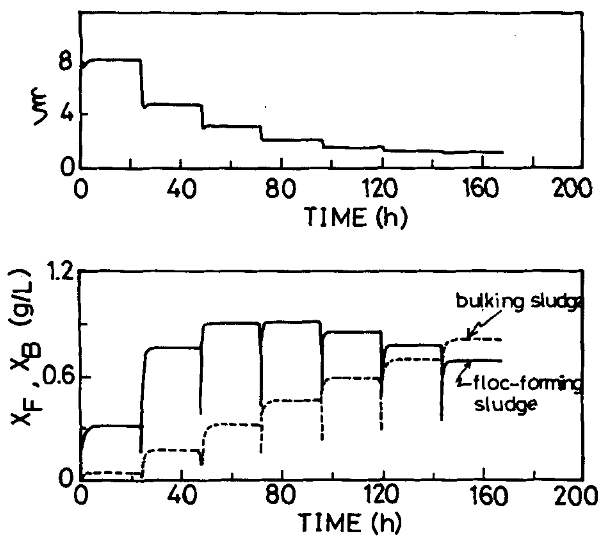

Figure 3. The simulation of fill-and-draw type batch culture using medium-B. After more than 4 days, the concentrated ratio ξ approached 1.0. After that, the bulking was observed.

Interaction between Bulking and Floc-Forming Sludges

It is assumed that there are no interactive kinetics between the bulking and floc-forming sludges. This could be partially clarified by the following experiment and computer simulation. When fill-and-draw-type batch experiments start from the state in which the ratio of the bulking sludge to the floc-forming sludge is very small, the bulking sludge increases gradually using standard medium-B, and settleability becomes worse after more than 4 days from the start, as stated previously. The initial COD concentration is about 600 $g \cdot m^{-3}$, and half the amount of mixed liquor is replaced by new medium in a day.

This experimental result can be simulated as shown in Figure 3 by the mathematical model given in Eq. 1 with no direct interaction between the bulking and floc-forming sludges. From Figure 3, the concentrated ratio ξ given by Eq. 2 is also shown, and after 4 or 5 days, ξ becomes almost 1.0, that is, the state of the sludge approaches bulking. This fact coincides with the experimental results. Then, the assumption that there is no direct interaction between the bulking and floc-forming sludges can be partially accepted as valid.

MATHEMATICAL MODEL OF THE ACTIVATED SLUDGE PROCESS

In an activated sludge process composed of a completely mixed aeration tank and a sedimentation vessel, the mathematical model of the system is derived as follows. The kinetic model described by Eq. 1 obtained in batch experiments is assumed to be valid in the aeration tank. It is also assumed that concentrations of the recycled bulking and floc-forming sludges X_F^r and X_B^r are given as; $X_F^r = \xi X_F$, $X_B^r = \xi X_B$, not considering the dynamic characteristics of the sedimentation vessel. The mathematical model of the activated sludge system considered here becomes,

$$\dot{X}_j = \{(\xi - 1)q_r - q_i + \mu_j\}X_j, \qquad j = F, B$$

$$\dot{S} = q_i(S_i - S) - \frac{\mu_F}{Y_F}X_F - \frac{\mu_B}{Y_B}X_B \qquad (3)$$

$$\mu_j = \begin{cases} K_j(S - S_j) & \text{for } S \geqq S_j \\ 0 & \text{for } S < S_j \end{cases} \qquad j = F, B$$

$$\xi = \alpha(X_B/X_F + \beta)^{-\gamma} + 1$$

where X_F, X_B, and S are outlet concentrations of floc-forming sludge, bulking sludge and COD from the aeration tank, and S_i is the inlet COD concentration of the system. And q_i and q_r are inlet flow rate and recycled flow rate divided by the tank volume, respectively. Equation 3 will be analyzed to obtain information about the bulking phenomenon caused by high organic loading. One of the important characteristics of Eq. 3 is that the growth rates of the bulking and floc-forming sludges intersect as shown in Figure 4. Using medium-B, the relationships of the specific growth rates of both sludges become similar to those using medium A.

The intersection defines the value of the COD concentration S_c at the coexistence of both types of sludge. As shown in Figure 4, in the region where COD is less than S_c, the specific growth rate of floc-forming sludge is larger than that of bulking sludge. That

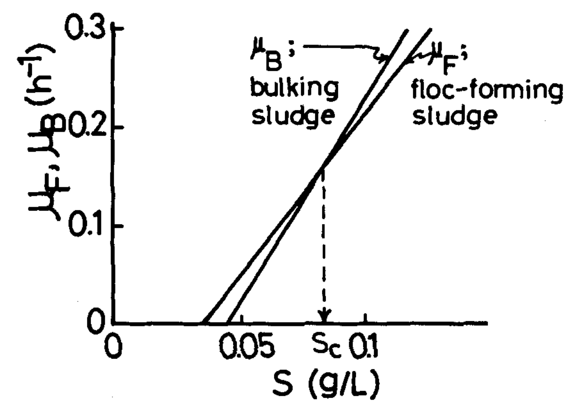

Figure 4. The specific growth rates of floc-forming sludge and bulking sludge using medium-A.

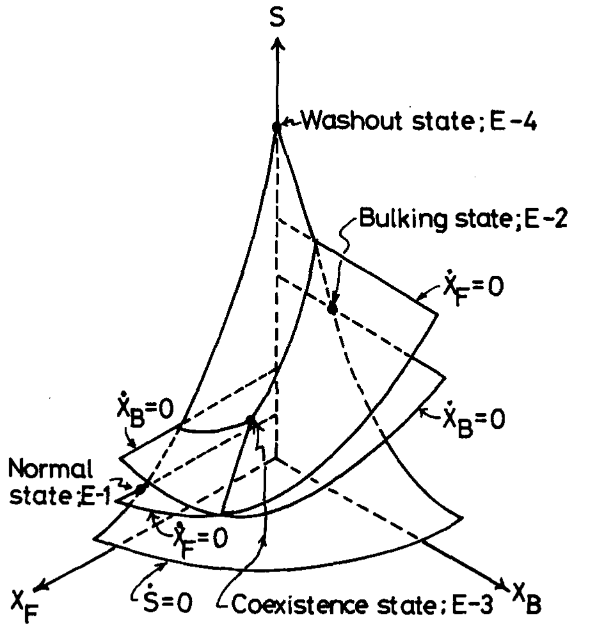

Figure 5. The possible equilibrium points which are shown as the intersection of the isocline and $X_F = 0$ or $X_B = 0$. Four possible points are shown as $E - 1$ to $E - 4$.

is, floc-forming sludge will possibly be dominant. On the other hand, in the region where COD is larger than S_c, the bulking sludge will be dominant.

Another distinguishable feature of the mixed culture system employed here, compared with one already reported (Stephanopoulos, 1980), is the existence of the cell recycle stream. Reflecting this feature, a coexistence state of both sludges should be taken into account in a certain range of the dilution rate, though the coexistence state does not exist without the cell recycle stream.

DYNAMICS OF AN ACTIVATED SLUDGE PROCESS

Equilibrium Points

The next problem to be analyzed is the dynamic behavior of Eq. 3 when the organic loading rates, S_i and q_i, are changed. As the characteristics of a mixed culture system using medium-B is similar to one using medium-A, analysis is based on the parameters obtained using medium-A. First, the possible steady states should be clarified.

It can be shown that there are, in general, four equilibrium points: normal state $E - 1(X_F \neq 0, X_B = 0)$; bulking state $E - 2(X_F = 0, X_B \neq 0)$; coexistence state $E - 3(X_F \neq 0, X_B \neq 0)$; and washout state $E - 4(X_F = 0, X_B = 0)$. Normal state $E - 1$ is the only desirable state for stable operation, and bulking state $E - 2$ should be avoided. Figure 5 shows the relationship of these four equilibrium points in the state space. These points are given as intersections of three planes, called "isoclines," and the plane $X_B = 0$ or $X_F = 0$. An isocline is defined here as the curve or the plane along which the first derivative of the state variables with respect to time t is zero excluding $X_B = 0$ and $X_F = 0$. Then, the isoclines are three planes which satisfy $\dot{X}_F = 0$, $\dot{X}_B = 0$ and $\dot{S} = 0$ in Eq. 3. From the definition, the intersections of these planes are equilibrium points. For example, point $E - 1$ is given as the intersection of both isoclines of $\dot{X}_F = 0$ and $\dot{S} = 0$, and the plane $X_B = 0$. The number of possible equilibrium points at a given operating condition will be counted using these isoclines, that is, the shape of these isoclines and their interconnection.

The trajectory of the state from a given initial condition to a final state will be roughly estimated using these three isoclines because an isocline divides the gradient of a state variable into two regions having negative and positive values. From the gradient of the three state variables, the gradient of the trajectory in the state space will be estimated.

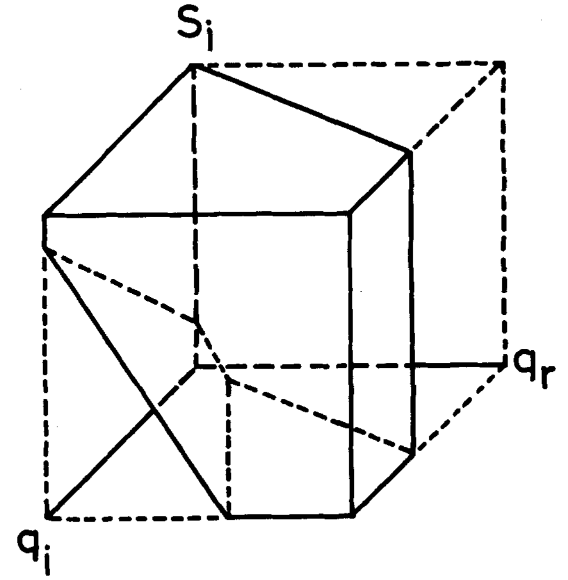

Figure 6. The domain of operating conditions in which $E - 1$ exists.

Finally, the maximum number of possible equilibrium points is four, Figure 5, and normal state $E - 1$ is the only desirable state for the stable operation of the activated sludge process. The other states should be avoided in the operation. To make it possible for $E - 1$ to exist, the necessary condition between operating conditions, such as input flow rate q_i, or recycled flow rate q_r and input COD concentration S_i, must satisfy the following equations:

$$S_F \leqq S_e \leqq S_i \tag{4}$$

where, $S_e \triangleq [q_i - (\xi^* - 1)q_r + K_F S_F]/K_F$

Where ξ^* is the value of ξ at $X_B/X_F = 0$. This constraint was deduced from the condition that normal state $E - 1$ in Figure 5 really exists. The condition can be reduced to be the interior of the region indicated in Figure 6. This analysis can be performed with respect to the other equilibrium points.

Locally Stable Conditions of the Activated Sludge Process

The plant should be operated at least on a condition guaranteeing that the equilibrium point $E - 1$ exists and is locally stable. The term "locally stable" means that the equilibrium point is asymptotically stable in the linearized system. The locally stable condition can be reduced to checking the sign of the real part of the eigenvalues of the characteristic equation in the linearized system (Athans and Falb, 1966). Stability is guaranteed only in the vicinity of the equilibrium point. Of course, in practical operation, whether the system goes to state $E - 1$ from a given initial state or not depends on the global stability based not only on local stability but also on the global dynamic behavior. However, locally stable conditions will give useful information for the qualitative strategy of stable operation. Using the locally stable condition, the domain of stable operation in the operating space consisting of S_i, q_i and q_r can be obtained.

The locally stable conditions of four equilibrium points were deduced analytically as the function of the operating conditions, S_i, q_i and q_r. It is proved that the coexistence state $E - 3$ is unstable under every kind of operating condition. As shown in Figure 7, the locally stable conditions and the existence conditions of the four equilibrium points divide the domain of the operating conditions into many regions. This figure can be used to identify the existence and stability of an equilibrium point.

Figure 8 shows the region described by q_i, q_r and S_i in which E

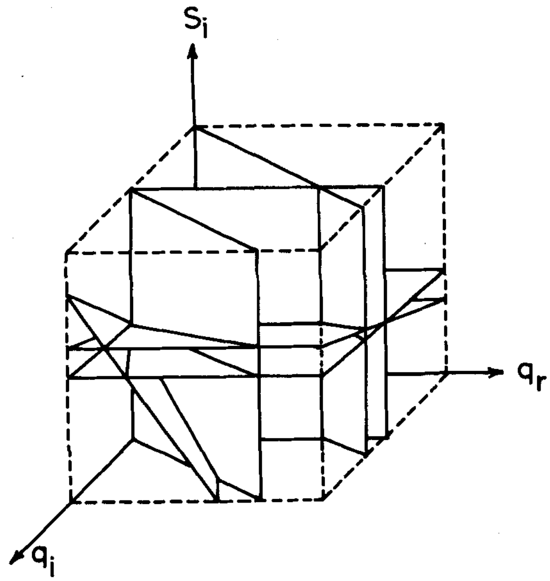

Figure 7. Locally stable conditions and those in which an equilibrium point exists divide the operating region into many domains.

$-$ 1 exists and is locally stable. Under the locally stable condition at $E - 1$, S_e defined in Eq. 4 should be in the following region,

$$S_F \leqq S_e \leqq S_c \tag{5}$$

where S_c is the intersection of μ_F and μ_B as shown in Figure 4. That is, $\mu_F \geqq \mu_B$ in this region, and floc-forming sludge becomes dominant. The well-known fact that high organic loading will cause a bulking phenomenon is verified using the model employed here, as shown in this figure. This figure can also be utilized to obtain the strategy for locally stable conditions. When the state doesn't change much from $E - 1$, the system will be operated in a stable way if the operating condition is maintained in this region. If the flow rate of waste water q_i increases, the recycled flow rate q_r should be increased, Figure 8, to maintain the conditions in this locally stable region.

Figure 9 shows the region which is the domain common to the region indicated in Figure 8 and the region in which the bulking equilibrium $E - 2$ is unstable or does not exist. In the domain of the operating conditions indicated in Figure 9, only $E - 1$ is locally stable. Even though the bulking sludge increases slightly, from the local stability it can be said that the desired operating state will be restored if the operating condition is maintained in this region. These conditions coincide with the well-known rule from experience that the sludge loading rate should be kept low. The kind of deviation in the operating and initial conditions that will make the normal state $E - 2$ unstable was clarified.

CONTROL OF AN ACTIVATED SLUDGE PROCESS PREVENTING THE BULKING PHENOMENON

The locally stable conditions are not sufficient to maintain a stable operation that prevents the bulking phenomenon. This is because in practice locally stable conditions, such as those shown in Figure 8, only guarantee stability in the vicinity of $E - 1$. Whether state $E - 1$ is reachable or not from a given initial state is not distinguishable because, in this region of the operating conditions, $E - 2$ is also locally stable. Moreover, the condition indicated in Figure 8 is obtained neglecting global dynamic behavior, i.e., neglecting the performance of the response. Also, the region which guarantees that only $E - 1$ is stable is too small in a practical sense, Figure 9.

Therefore, the manipulating variables should be changed dynamically to keep the system at normal state $E - 1$ when disturbances have been introduced. Next, the problem of how to regulate the state at $E - 1$ by manipulating the recycle flow rate q_r with disturbances is considered.

First, the state feedback control with linear dynamics and quadratic performance criteria, known as the LQP-problem (Kalman, 1963; Athans and Falb, 1966), is introduced. The nonlinear system described by Eq. 3 is linearized around $E - 1$, and the LQP regulator is introduced. More detailed explanation of the LQP regulator is given in the Appendix. One of the results of the simulation is shown in the $X_B - X_F$ phase plane of Figure 10. When the initial disturbances are imposed, the LQP regulator is not effective in preventing the system from going to bulking state $E - 2$ as in the nocontrol case.

However, it is shown that a type of nonlinear state feedback regulator proposed by Izawa and Hakomori (1980) makes the system stable; this maintains it at the nominal state $E - 1$. If the

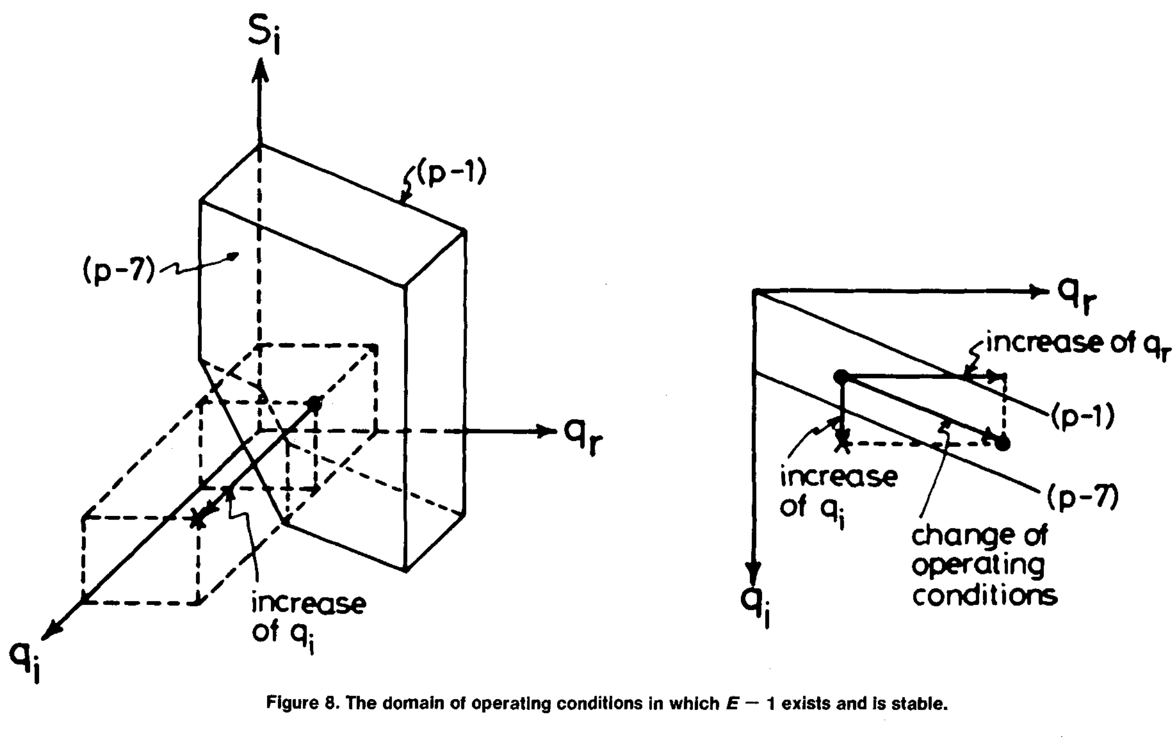

Figure 8. The domain of operating conditions in which $E - 1$ exists and is stable.

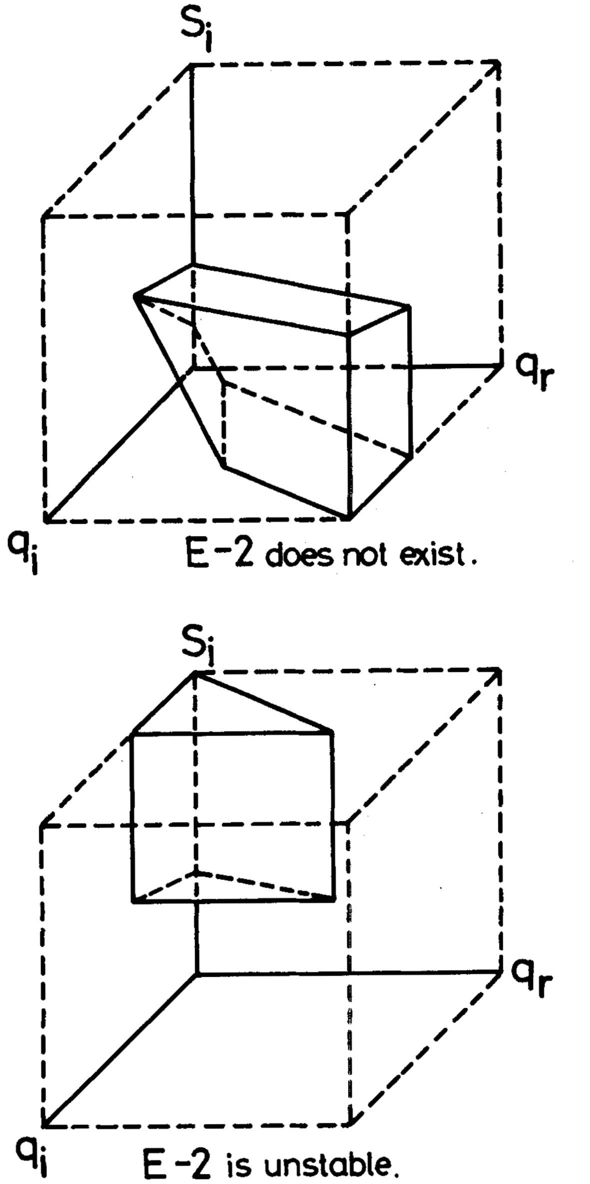

Figure 9. The desirable operating domain so that $E - 1$ is stable and the other equilibrium points don't exist or are unstable.

trajectory of the linear system is observed using a curved coordinate system, the trajectory of the observed system becomes one obtained from a nonlinear system. Then, a nonlinear system becomes equivalent to a linear system if an appropriate curved coordinate is used. From this concept, the condition of the transformation from

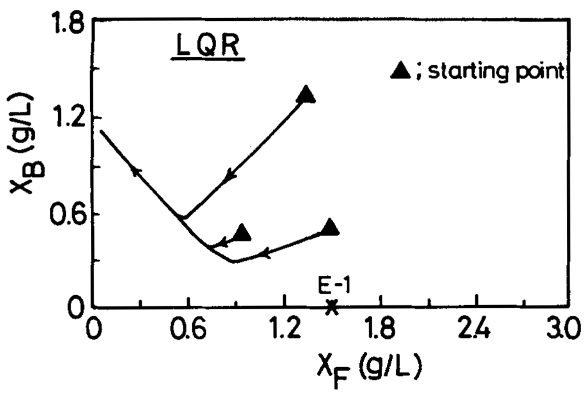

Figure 10. LQP control to prevent bulking phenomenon.

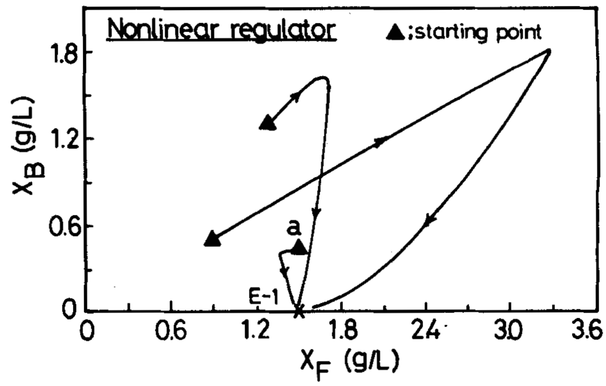

Figure 11. A nonlinear regulator to prevent bulking phenomenon.

a linear to nonlinear system is clarified. A more detailed explanation is given in the Appendix. Finally, this nonlinear state feedback regulator is introduced into the nonlinear system described by Eq. 3 as shown in the Appendix.

Figure 11 shows the result using the nonlinear regulator obtained by the computer simulation. When initial disturbances are imposed, for example, from point a the state can be regulated to normal state $E - 1$ with the nonlinear regulator despite the fact that the system goes to bulking state $E - 2$ using the LQP regulator or with no-control. By comparing Figures 10 and 11, it is seen that the nonlinear regulator works well and that the normal state where the floc-forming sludge is dominant can be maintained well. Figure 12 shows the time course of the state variables with the nonlinear regulator starting from point a in Figure 11. In the initial few hours q_r should be greatly changed, and the state will gradually return to normal state $E - 1$.

Finally, a nonlinear state feedback regulator can control the activated sludge preventing the bulking phenomenon. However, we still have a problem in applying the theory to a real plant. It is how to detect or measure the concentration of bulking sludge or floc-forming sludge.

CONCLUDING REMARKS

It has been shown that the simple mathematical model of the mixed culture describes the bulking phenomenon caused by high sludge loading in the activated sludge process well. The mathematical model consisted of two groups of microorganisms: floc-forming sludge and bulking sludge. The description of the kinetics

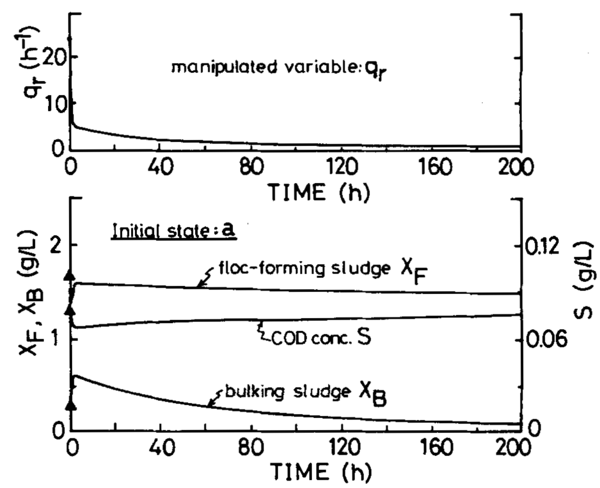

Figure 12. One example of the time-course of X_F and X_B due to a nonlinear regulator.

in the aeration tank and of settleability in the sedimentation vessel were modeled from the experimental study. Using the mathematical model, the conditions that will cause the bulking phenomenon were clarified. In particular, it should be stressed that the conditions under which high organic sludge loading causes the bulking phenomenon can be explained mathematically in this system. The strategy preventing bulking phenomenon was discussed using the mathematical model. Also, the nonlinear state-feedback regulator applied here can control the bulking sludge.

To apply the result to a real system, there still is a problem that should be solved. With respect to the mathematical model, the kinetics of a tubular-type reactor and the dynamic model of sedimentation should be discussed along with whether the kinetics obtained from batch culture should be applicable to practical cases or not. With respect to the control problem, the way to detect or measure the state of the activated sludge and how to build the control system should be discussed further as well.

NOTATION

K_j = kinetic parameter given in Eq. 1 ($j = B, F$)[$h^{-1}\cdot kg^{-1}\cdot m^3$]
q_i = inlet flow rate divided by the tank volume [h^{-1}]
q_r = recycled flow rate of sludge divided by the tank volume [h^{-1}]
S_i = inlet COD concentration [$kg\cdot m^{-3}$]
S = COD concentration [$kg\cdot m^{-3}$]
S_e = given in Eq. 4 [$kg\cdot m^{-3}$]
S_j = parameters given in Eq. 1 ($j = B, F$) [$kg\cdot m^{-3}$]
X_j = outlet concentration of jth sludge ($j = B, F$) [$kg\cdot m^{-3}$]
Y_j = yield factor ($j = B, F$)

Greek Letter

α = parameter in Eq. 2
β = parameter in Eq. 2
γ = parameter in Eq. 2
ξ = concentrated ratio defined as the settled sludge concentration against the concentration to be settled
μ_j = specific growth rate [h^{-1}]
ν_j = specific COD uptake rate ($j = B, F$) [h^{-1}]

Subscripts & Superscripts

B = bulking sludge
F = floc-forming sludge
r = recycled sludge

APPENDIX

For ease of explanation, it is assumed that the original system equation is written as,

$$\dot{x} = f(x, q_r) \tag{A1}$$

instead of Eq. 3, where state variable x is given as $x^t \triangleq (X_F, X_B, S)$. Now, consider that the deviations of each state variable x and control variable q_r from x^* at the equilibrium point $E - 1$ and q_r^* are given as δx and δu, that is,

$$\Delta x \triangleq \begin{pmatrix} \delta x_1 \\ \delta x_2 \\ \delta x_3 \end{pmatrix} \triangleq \begin{pmatrix} X_F - X_F^* \\ X_B - X_B^* \\ S - S^* \end{pmatrix} \quad \delta u \triangleq q_r - q_r^* \tag{A2}$$

Substituting Eq. A2 into Eq. A1 and using the first order approximation of Taylor's series at $E - 1$, the nonlinear system is approximately expressed by the following linear system,

$$\delta \dot{x} = C\delta x + D\Delta u \tag{A3}$$

Where 3 × 3 dim. matrix C and 3 × 1 dim. matrix D are,

$$C \triangleq \frac{\partial f}{\partial x} \qquad D \triangleq \frac{\partial f}{\partial q_r}$$

and these derivatives are evaluated at x^* and q_r^*.

The first problem considered here is to regulate δx by manipulating δu. It is necessary to find the control law that optimizes performance where a controlled process is linear, represented by Eq. A3, and a performance criteria to be minimized is specified as,

$$J = \frac{1}{2}\int_0^\infty (\delta x^t Q \delta x + \delta u^2)dt \tag{A4}$$

where in the numerical calculation Q is taken as a unit matrix. Such problems with linear dynamics and quadratic performance criteria are known as LQP-problems. In the LQP-problems, the optimal regulator implies the linear control law (Kalman, 1963)

$$\delta u = -D^t \cdot P \cdot \delta x \tag{A5}$$

where P is a solution of the Riccati's equation as follows,

$$PDD^tP - PA - A^tP - Q = 0 \tag{A6}$$

Superscipt t is the action of transposing the matrix.

Next, consider the nonlinear state feedback regulator. Substituting Eq. A2 into Eq. A1 and rearranging it using $\dot{x}^* = 0$ and $f(x^*, q_r^*) = 0$, the following equation represented by δx and δu,

$$\delta x = a(\delta x, \delta u)\delta x + b(\delta x, \delta u)\delta u \tag{A7}$$

is obtained where a and b are 3 × 3 and 3 × 1 matrices, respectively. Equation A7 is nonlinear in spite of the fact that Eq. A3 is linear. The problem considered here is one of obtaining an optimal nonlinear regulator $\delta u = \delta u(\delta x, t)$ with Eq. A7 and performance criteria A4 to be minimized. Without reducing the problem into a two-point boundary value problem, the optimal regulator problem is solved by the method proposed by Izawa and Hakomori (1980).

The method consists of the following steps: (1) finding a fictious linear system which is homeomorphic to a given nonlinear plant; (2) designing an optimal state regulator for the above linear system; and (3) constructing the nonlinear state regulator having the same topological structure as the above designed optimal state regulator. Control laws are derived in terms of the homemorphism. To describe the homeomorphism, a Riemannian geometric model, based on the idea that in the state space a nonlinear system referres to appropriate curvilinear coordinates will behave as a linear system, is developed. Equations which determine the curvilinear coordinate system are also derived.

Based on the method described above, our problem can be solved as follows: First, consider the following linear system similar to Eq. A7,

$$\dot{X} = AX + BU \tag{A8}$$

where A and B are 3 × 3 and 3 × 1 matrices evaluated at the initial condition such that,

$$A \triangleq a(\delta \times (0), \delta u(0)),$$
$$B \triangleq b(\delta x(9), \delta u(0)).$$

With this linear system given by Eq. A8, an optimal regulator is synthesized as,

$$U = -KX \tag{A9}$$

where K is a 1 × 3 dim. matrix.

Now, the nonlinear system (Eq. A7) and the linear system (Eq. A8) become homeomorphic according to the following mapping τ.

$$\tau \triangleq \begin{pmatrix} \tau_{11}\tau_{12} \\ \tau_{21}\tau_{22} \end{pmatrix} \tag{A10}$$

where τ_{11}; 3 × 3 dim: τ_{12}; 3 × 1 dim.: τ_{21}; 1 × 3 dim.: τ_{22}; 1 × 1 dim. matrices, respectively.

Optimal control law δu can be obtained as,

$$\delta u = -(\tau_{22} + K\tau_{12})^{-1}(\tau_{21} + K\tau_{11})\delta x \quad \text{(A11)}$$

using Eqs. A9 and A10. The transformation matrix τ is governed by the following differential equation,

$$\frac{d\tau}{dt} = \begin{pmatrix} A, B \\ 0, 0 \end{pmatrix} - \begin{pmatrix} a(\delta x, \delta u), (\delta x, \delta u) \\ 0, \quad 0 \end{pmatrix} \quad \text{(A12)}$$

with the initial condition $\tau(0) = I$ (unit matrix).

In accordance with this method, $a(\delta x, \delta u)$ and $b(\delta x, \delta u)$ are taken as follows in the numerical example:

$$a_{11} = [(\xi - 1)q_r^* - q_i + k_F(\delta x_3 + S^* - S_F) - \delta u](1 + X_F^*/\delta x_1)$$

$$a_{12} = a_{13} = a_{21} = 0$$

$$a_{22} = [(\xi - 1)q_r^* - q_i + k_B(\delta x_3 + S^* - S_B) - \delta u](1 + X_B^*/\delta x_2)$$

$$a_{23} = 0$$

$$a_{31} = -\frac{k_F}{Y_F}(\delta x_3 + S^* - S_F)(1 + X_F^*/\delta x_1)$$

$$a_{32} = -\frac{k_B}{Y_B}(\delta x_3 + S^* - S_B)(1 + X_B^*/\delta x_2)$$

$$a_{33} = -q_i$$

$$b_1 = \xi(\delta x_1 + X_F^*)$$

$$b_2 = \xi(\delta x_2 + X_B^*)$$

$$b_3 = 0$$

The control law using the nonlinear state feedback regulator is calculated by Eqs. A8 to A12.

LITERATURE CITED

Aris, R., and A. E. Humphrey, "Dynamics of a Chemostat in Which Two Organisms Compete for a Common Substrate," *Biotechnol. & Bioeng.*, **19**, p. 1375 (1977).

Athans, M., and P. L. Falb, *Optimal Control: An Introduction to the Theory and Its Application*, McGraw-Hill (1966).

Attir, U., and M. M. Denn, "Dynamics and Control of the Activated Sludge Wastewater Process," *AIChE J.*, **24**, p. 693 (1978).

Chudova, J., V. Ottova, and V. Madera, "Control of Activated Sludge Filamentous Bulking. I: Effect of the Hydraulic Regime or Degree of Mixing in an Aeration Tank," *Water Research*, **7**, p. 1163 (1973a).

Chudova, J., P. Grau, and V. Ottova, "Control of Activated Sludge Filamentous Bulking. II: Selection of Microorganisms by Means of a Selector," *Water Research*, **7**, p. 1389 (1973b).

Chudova, J., J. Blaha, and V. Madera, "Control of Activated Sludge Filamentous Bulking. III: Effect of Sludge Loading," *Water Research*, **8**, p. 231 (1974).

Dick, R. I., "Role of Activated Sludge Final Settling Tanks," *Proc. of ASCE*, Sanitory Eng. Div., **SA2**, p. 423 (1970).

Diviasio, D., H. C. Lim, W. A. Weigand, and G. T. Tsao, "Phase-plane Analysis of Feedback Control of Unstable Steady States in a Biological Reactor," *AIChE J.*, **24**, p. 686 (1978).

Diviasio, D., H. C. Lim, and W. A. Weigand, "An Experimental Investigation of Stability and Multiplicity of Steady States in a Biological Reactor," *AIChE J.*, **27**, p. 284 (1981).

Ford, D. L., and W. W. Eckenfelder, Jr., "Effect of Process Variables on Sludge Floc Formation and Settling Characteristics," *J. Water Poll. Control Fed.*, **39**, p. 1850 (1967).

Houtmeyers, J., E. van den Eynde, R. Poffe, and H. Verachtert, "Relations Between Substrate Feeding Pattern and Development of Filamentous Bacteria. Part 1: Influence of Process Parameters," *European J. Appl. Microbiol. and Biotechnol.*, **9**, p. 63 (1980).

Izawa, Y., and K. Hakomori, "Design of Nonlinear Regulators Using Riemannian's Geometric Model," *Trans. of Soc. of Instrument and Control Engineers Japan*, **16**, p. 628 (1980).

Kalman, R. E., *The theory of Optimal Control and the Calculus of Variations*, Chap. 16, R. Bellman, Ed., University of California Press (1963).

Palm, J. C., D. Jenkins, and D. S. Parker, "Relationship Between Organic Loading, Dissolved Oxygen Concentration and Sludge Settleability in the Completely-mixed Activated Sludge Process," *J. Water Poll. Control Fed.*, **52**, p. 2484 (1980).

Rensink, J. H., "New Approach to Preventing Bulking Sludge," WPCF, **46**, p. 1888 (1974).

Stephanopoulos, G., A. G. Frederickson, and R. Aris, "The Growth of Competing Microbial Populations in a CSTR with Periodically Varying Input," *AIChE J.*, **25**, p. 863 (1979).

Stephanopoulos, G., "Dynamics of Mixed Cultures of Microorganisms; Some Topological Considerations," *AIChE J.*, **26**, p. 802 (1980).

Unno, H., and T. Akehata, "Effect of Filamentous and Nonfilamentous Sludges on the Settling Curve," 14th Fall Meeting of SCEJ, p. 765 (1980).

Verachtert H., E. Van den Eynder, R. Poffe, and J. Houtmeyers, "Relations Between Substrate Feeding Pattern and Development of Filamentous Bacteria. Part 2: Influence of Substrates Present in the Influent," *European J. Appl. Microbiol. and Biotechnol.*, **9**, p. 137 (1980).

Yoon, H., and H. W. Balanch, "Competition for Double Growth Limiting Nutrients in Continuous Culture," *J. Appl. Chem. Biotech.*, **27**, p. 260 (1977).

Yoshida, T., B. S. Rao, S. Ohasa, and T. Taguchi, "Dynamic Analysis of a Mixed Culture in Chemostat," *J. Ferm. Technol.*, **56**, p. 546 (1979).

Manuscript received July 30, 1982; revision received April 28, and accepted June 10, 1983.